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Uncertainties on the measurements of the top mass at a future e^+e^- collider

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Abstract. The uncertainties due to limited knowledge of the multi-hadron final state on the measurements of the top mass at future linear colliders are discussed. This study is performed for $e^+e^- \to t\bar{t}$ annihilation events at the center-of-mass energy of $s^{1/2} = 500\,\mathrm{GeV}$ using Monte Carlo models tuned to LEP experiments. The uncertainties are determined for the all-hadronic top-decay mode as well as for the lepton-plus-jets channel.

1 Introduction

According to the standard model (SM), the top quark is the heaviest quark known, which has a mass intriguingly close to the scale of electroweak symmetry breaking. The mass of the top quark, m_t, being one of the most fundamental parameters of the SM, allows one to test the consistence of the SM and can be used to predict unknown SM parameters. For example, with a precise measurement of the top mass, together with an accurate determination of the W boson mass, M_W, an indirect constraint on the mass of the Higgs boson can be obtained.

Several properties of the top quarks have already been measured at the Tevatron. In particular, the combined result from the Tevatron experiments gave $m_t = 174.3 \pm 3.2(\mathrm{stat}) \pm 4.0(\mathrm{syst})\,\mathrm{GeV}$ [1]. At the LHC experiments, the measurements of m_t are expected to be feasible with a precision of better than $2\,\mathrm{GeV}$ [2], although there are indications that for some statistically non-dominant decay channels the measurements might have a systematic uncertainty of $\sim 1\,\mathrm{GeV}$ [3].

The top physics will be one of the main interests at future linear e^+e^- colliders. Clean experimental conditions of the process $e^+e^- \to t\bar{t}$ would allow one to determine the top mass and its width with unprecedented precision. With the large rate of top events anticipated (about 150 000 $t\bar{t}$ pairs for a linear collider operating at $s^{1/2} = 500\,\mathrm{GeV}$ with an integrated luminosity of $\sim 200\,\mathrm{fb}^{-1}$ per year), the uncertainty on the reconstructed mass will be dominated by theoretical and experimental systematical errors.

A detailed assessment of theoretical errors has to take into account the uncertainties due to different methods used in the next-to-next-to-leading order QCD correction calculations. Such uncertainties can lead to an error on the $\overline{\mathrm{MS}}$ top mass of $\sim 100\,\mathrm{MeV}$ [4]. The top-mass measurements based on the reconstruction of the invariant mass of jets originating from top quarks should be considered as determinations of the top-quark pole mass. The latter mass definition, which is currently used in Monte Carlo (MC) models, has a limitation on the accuracy; the extraction of the top-quark pole mass has a theoretical uncertainty of around 300 MeV [4] and cannot be determined with a precision better than $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ [4,5]. Moreover, when the multi-hadronic final state is used in the reconstruction of the top quarks, such a precision on the pole mass may not be achievable due to limited knowledge on high-order QCD gluon radiations, determining the gluon activity in events used in the reconstruction, assumptions concerning the non-perturbative region of QCD, where the gluons and quarks are transformed into hadrons, as well as due to other hadronic final-state phenomena to be discussed below.

In this paper we study the precision on the top-quark pole mass attainable at future linear e^+e^- colliders operating at $s^{1/2} = 500\,\mathrm{GeV}$, concentrating on the multiparticle QCD aspects of the top decays. Presently, the multi-hadron production phenomena cannot be derived solely from perturbative QCD theory without additional model-dependent assumptions. Therefore, this analysis is based on Monte Carlo models, which are the only tools which allow us to study the multi-hadronic phenomena and their impact on the reconstructed observables in a systematic way, since these models provide a complete and detailed description of all known stages of the multiparticle production.

2 Multihadronic aspects of top decays

The top quarks decay almost exclusively via $t \to Wb$, thus the final-state topology of $t\bar{t}$ events essentially depends on the decay modes of the W bosons, which can decay either

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hadronically ($W^\pm \to q_1\bar{q}_2$) or via the leptonic channel ($W^\pm \to l^\pm\nu$). In this paper we analyze the following top decays which are statistically dominant at e^+e^- colliders:

$$e^+e^- \to t\bar{t} \to b\bar{b}W^+W^- \to b\bar{b}q_1\bar{q}_2q_3\bar{q}_4 \to 6 \text{ jets}, \quad (1)$$

$$e^+e^- \to t\bar{t} \to b\bar{b}W^+W^- \to b\bar{b}l\nu q_1\bar{q}_2 \to \text{lepton} + 4 \text{ jets}. \quad (2)$$

The first process arises in 44.4% of all $t\bar{t}$ decays, and is characterized by the presence of six jets in the final state ("fully hadronic" or "the all-hadronic" channel). This decay suffers from a background from QCD multi-jet events, which can be rather large at the LHC experiments. For e^+e^- annihilation events, this problem is expected to be less actual for an efficient double b-tagging. The Tevatron experiments have shown that it is possible to isolate $t\bar{t}$ production in this decay mode, despite the very complicated hadronic final state of the $p\bar{p}$ collisions.

The second process (2) is characterized by the presence of a high p_{T} lepton, four hadronic jets and the missing momentum of an unmeasured neutrino produced in the leptonic W decay ("semi-leptonic" or "lepton-plus-jets" channel). For such events, the neutrinos from the decay $W^\pm \to l^\pm\nu$ can be reconstructed using the energy-momentum conservation, since the $t\bar{t}$ decays are kinematically constrained. This decay channel has lower statistics (29.6% of all $t\bar{t}$ decays), however, because of the well-reconstructed high p_{T} lepton, one could significantly suppress the multi-jet QCD background.

In this paper, we study the impact of various, not well understood effects related to the multi-hadronic final state on the direct measurements of m_t in processes (1) and (2).

2.1 Multiple gluon radiations

Soft partons resulting from the hard subprocess undergo successive branchings. Such emissions play a significant role in building up the event structure. At present, however, complete perturbative calculations are not available, and only the parton-shower approach implemented in various MC models allows one to describe an arbitrary number of gluon branchings by simplifying the underlying dynamics of the multiple-gluon radiations. There are a few approaches to deal with this stage within the framework of MC models, which can have different implementations of the ordering in the coherent gluon emissions. The HERWIG model [6] orders the emissions in angle, while the PYTHIA model [7] orders them in decreasing invariant mass with an additional constraint to ensure the angular ordering. The ARIADNE model [8] orders the parton emissions in the transverse momentum.

It is not possible at this moment to say which approach is the best; they all reflect different aspects of the QCD multi-parton dynamics in the parton-shower approximation. Experimentally, the major features of e^+e^- events are rather similar for all well-tuned MC models [9]. However, insignificant discrepancies between these models for the LEP experiments could have a dramatic effect on the future high-precision measurements at a larger center-of-mass energy of e^+e^- collisions, which obviously implies a stronger contribution from the gluon showering at the perturbative QCD stage.

The reconstruction of jets in the processes (1) and (2) requires the use of jet finding algorithms, which are at present indispensable tools in organizing the sprays of hadrons (partons) into some number of jets. For the identification of the massive particles, they help to reconstruct the momenta of the initial quarks originating from the hard subprocesses and allow a separation to be made of perturbative and non-perturbative QCD regions. Since an exact definition of a resolvable jet is needed not only on the experimental (hadronic) level, but also on the theoretical (partonic) level, it is mandatory to consider the theoretical approaches to the multiple parton radiation together with a particular definition of the jet algorithm. Jet algorithms use different criteria for combining particles into jets, thus they all suffer from misclusterings in a different degree. This introduces an additional uncertainty on the determination of the top mass.

In this paper, a few most popular jet clustering algorithms are used: DURHAM [10], JADE [11] and LUCLUS [12]. At present, there is no a unique criterion for the best algorithm; they all perform comparably well for a large distance measure. The success of the JADE algorithm in the reconstruction of multi-jet events and the W mass is less evident than for the algorithms based on an p_{T} distance measure [13]; nevertheless, we will include the results with the JADE algorithm for completeness.

2.2 Jet fragmentation and related non-perturbative effects

The subsequent parton cascade is followed by a soft fragmentation process. The latter occurs with small momentum transfers which may be considered to extend to a value Q_0, which is a QCD cut-off above which perturbative methods can be applied. Note that this unnatural cut-off used in Monte Carlo models could produce non-perturbative model-dependent distortions already for the parton predictions (the so-called "parton level") of these models.

The hadronization stage itself is not well understood from first principles, and thus it is a subject of important uncertainties. The hadronization mechanism can be simulated using the Lund string model as implemented in PYTHIA/JETSET [7] and ARIADNE [8]. In HERWIG, the hadronization is described by the cluster fragmentation model [14].

Further, when heavy particles like t-quarks are produced in pairs and decay, the hadronic systems overlap during the fragmentation process. This occurs because the typical decay distance, determined by the decay width of these particles, is smaller than the typical hadronic scale $\mu \sim 1\,\mathrm{fm}^{-1}$. Therefore, a high-precision reconstruction of the t masses is non-trivial as it requires the understanding of non-perturbative, long-distance QCD effects caused by a large overlap between the hadronic decay products of W and t-quarks. For example, it is well known that the color reconnection (CR) [15–18] and the Bose–Einstein (BE) effect [19,20] can produce systematic uncertainties on the W and t mass measurements when the hadronic final state is used in the reconstruction.

While the results from the LEP2 experiments are not conclusive with respect to the significance of these two effects [21,22], at a future linear collider the situation may change. Considering that the future experiments will study the e^+e^- annihilation events with a significantly larger luminosity, aiming to study the W and t masses with a high precision, it is important to understand how strongly such measurements could be affected by the CR and BE effects.

In addition to the effects discussed above, variations in the hadronic composition of jets, in production rates of heavy resonances and in the fraction of neutrinos escaping detection can all alter the details of the hadronic final state. Such effects are especially important for the top production, the underlying physics of which involves large production rates of beauty and charm particles and represent a real challenge for the MC models in use.

One of the sources of uncertainty in the measurement of the top quarks at the Tevatron is the b-fragmentation [23], which is usually described using the Peterson fragmentation function. The parameter ϵ_b of this parameterization varies within a large range for different experiments [9]. We will consider as realistic values of ϵ_b between 0.002 and 0.006, following [24,9]. In addition, as an alternative to the Peterson fragmentation, we will study the LUND-based string fragmentation model for heavy flavor production included in the default setting of PYTHIA.

In this paper we will not consider the systematical effects arising from the QED bremsstrahlung, concentrating only on the less understood hadronic aspect of the top decay. The systematics due to uncertainties on the W mass are also outside the scope of this paper.

At present, the study of the effects discussed above can only be performed using Monte Carlo models which have many free parameters. To determine the uncertainties arising from intrinsic ambiguities in their values, modifications of such parameters have to be done in a reasonable ("physical") range. By doing this, however, a realistic estimate for the uncertainties is difficult to obtain: many MC parameters correlate and a MC model with only one modified parameter is likely to be unable to reproduce the existing e^+e^- data because a specific MC tuning might be destroyed.

In this paper, we will adopt the following approach: instead of variations of MC parameters responsible for a particular stage of the multi-jet production, we will use various MC tunings from the LEP experiments. First of all, this would allow us to consider a meaningful range of values for MC parameters. On the other hand, we will stay within a particular MC tuning, not distorting agreements between MC models and e^+e^- annihilation data.

3 Top-mass reconstructions

In this paper we are not aiming to suggest a particular approach for the top reconstruction, but rather will use the most simple methods which are sufficient for the purposes of this article. A realistic detector simulation as well as studies of the background processes are required to understand the applicability of the methods described below.

The MC events for processes (1) and (2) were generated at the center-of-mass energy of $s^{1/2} = 500\,\mathrm{GeV}$. We use the most recent versions of MC models: PYTHIA 6.2 [7], HERWIG 6.4 [6] and ARIADNE 4.12 [8]. The nominal value of the W mass was $M_W = 80.45\,\mathrm{GeV}$ and the Breit–Wigner width was set to 2.071 GeV (these values correspond to the PYTHIA 6.2 default settings). The mass of the generated top quarks was $m_t = 175.0\,\mathrm{GeV}$ and the corresponding width of the Breit–Wigner distribution was set to 1.398 GeV. In the HERWIG model, the top-quark width cannot be simulated. The final-state particles (hadrons, photons and leptons) with lifetime $c\tau > 15\,\mathrm{cm}$ were considered as stable.

3.1 Fully hadronic $t\bar{t}$ decay

As a first step, a jet algorithm was applied to reconstruct the four-momenta of jets in the process (1). All particles were grouped to exactly six jets, thus allowing for a distance measure y_{cut} of the jet algorithms to have different values for every event. Events were accepted if all reconstructed jets have transverse momenta above 10 GeV.

The double b-tagging is assumed throughout this paper. This allows us to distinguish between light-flavored jets and b-quark jets, thus helping to reduce the combinatorial background and to simplify the reconstruction. To identify the b-quark jets, we match the four-momenta of the generated b-quarks to the momenta of reconstructed jets using a cone algorithm with a radius of 0.5 in the pseudorapidity and the azimuthal angle of jets.

The jets which are not tagged as b-jets were used to reconstruct the dijet invariant mass, M. For a W candidate, we require for the dijet mass to be within the mass window $\mid M_W - M \mid < 5\,\mathrm{GeV}$, where M_W is the nominal mass of the W bosons. An event was accepted if exactly two W candidates were found. (Below we will discuss a more complicated method which is better suited for the experimental conditions.) For the accepted events, two W candidates were combined with b-tagged jets to form the invariant mass of top (anti-top) candidates.

The top mass and width were determined from the fit procedure using the Breit–Wigner distribution together with a term describing the combinatorial background. The object-oriented data analysis framework ROOT [25] was used for the fits. For the background, we use the quadratic polynomial form, $a+bM+cM^2$ (with a, b, c being the free parameters). Note that the choice of the best fit function is not trivial, and the chosen parameterization might be inappropriate for the realistic experimental reconstruction in which a convolution of the Breit–Wigner function with a Gaussian distribution is required to describe the detector resolution, QED initial-state smearing, limited detector acceptance, etc.

There is another difficulty in the studies of top-quark events: many neutrinos from the b-quark fragmentation escape without detection. To deal with this problem, energy-momentum conservation constraints can be imposed to remove events with a significant fraction of neutrinos:

$$\left|\frac{E_{\mathrm{vis}}}{\sqrt{s}}-1\right|<0.03,$$
$$\frac{|\sum_i p_{||i}|}{\sum_i |\vec{p}_i|}<0.03, \quad \frac{\sum_i p_{\mathrm{T}i}}{\sum_i |\vec{p}_i|}<0.03, \qquad (3)$$

where E_{vis} is the visible energy, and $p_{||i}$ and $p_{\mathrm{T}i}$ are the longitudinal and the transverse momentum of a final-state particle.

While the restrictions (3) are essentially irrelevant for the parton-level studies to be discussed below, they are rather tight for the hadron level. This simplification helps to reject events with a large missing momentum/energy leading to asymmetric tails of the mass distributions for the final reconstruction of the top quarks. After the requirement (3), the simple fit discussed above can be used to extract the mass of the top quarks. Such a simplification, however, is unnecessary for a more sophisticated fit function in the realistic experimental reconstruction procedure.

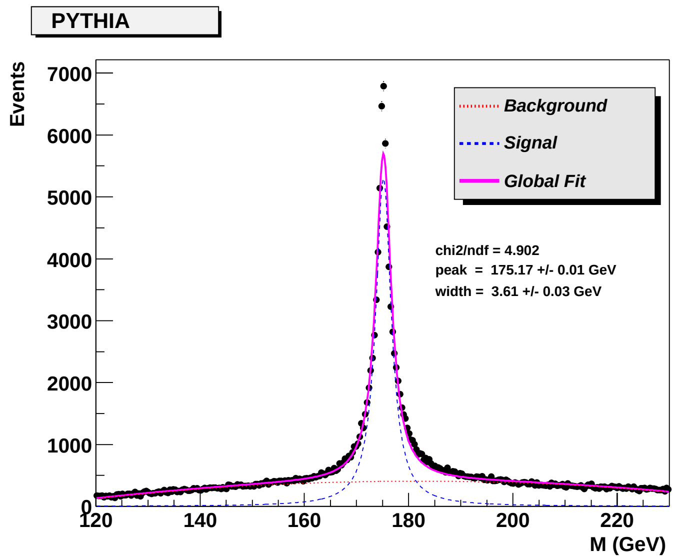

Fig. 1. The invariant-mass distribution used to reconstruct the top candidates in the fully hadronic $t\bar{t}$ decays. The parton level of the PYTHIA model with the default parameters was used

3.2 Semi-leptonic $t\bar{t}$ decay

In the case of process (2), it is necessary to reconstruct exactly four jets, in addition to a high p_{T} lepton. We use only events with $E_{\mathrm{T}} > 10\,\mathrm{GeV}$ for the reconstructed jets, and require the transverse momentum p_{T} of the detected lepton to be above 10 GeV. The kinematics of the decay mode (2) is fully constrained, like in the case of fully hadronic $t\bar{t}$ decays. The missing energy and momentum have been assigned to a neutrino escaping detection, therefore, no any cuts similar to (3) were imposed. The double b-tagging is used as before.

For the semi-leptonic decays, one W candidate can be reconstructed from the momenta of the lepton and neutrino, while the second W can be obtained from the invariant-mass distributions of jets which do not belong to the b-initialized jets. However, reconstructing the top candidates, only the W candidates obtained from the lepton and neutrino were used, requiring the invariant mass M of the W candidates to be within the mass window $|M_W - M| < 5\,\mathrm{GeV}$. We do not use the hadronically decaying Ws for the top-quark reconstruction due to the following reason: This case is completely identical to the all-hadronic decays and, therefore, it is less interesting when comparing the semi-leptonic decays with the fully hadronic top-decay mode.

4 Parton-level study

In this section, we will consider the reconstruction of top quarks from partons (photons) radiated by the quarks after the hard subprocess. The multiple-gluon radiation plays the key role in building up the structure of the top-quark events; therefore, there are uncertainties in how the basic properties of the multi-partonic system are described. As was noted before, there exist differences between Monte Carlo implementations of this stage; moreover, even within the scope of one particular MC model, there are sizable uncertainties in the values of tunable MC parameters used to model this stage.

The reconstruction of top quarks from the partons proceeds through the steps discussed in the previous sections. As an illustration, Fig. 1 shows the invariant mass of top candidates in the process (1) for the PYTHIA model with the default parameters. The solid thick line shows the Breit–Wigner fit function together with the background parameterization. The fit function is not well suited for the sharp peak near the nominal top-mass value; this drawback, however, gives a negligible effect for the results discussed below[1].

The reconstructed top masses (determined from the peak values of the Breit–Wigner fit) and widths are given in Figs. 2 and 3, respectively. We use the Durham jet algorithm as the default for PYTHIA with various LEP tunings (L3, ALEPH and OPAL settings [9]), for ARIADNE (DELPHI and ALEPH tunings [9]) and for HERWIG (with the OPAL tuning [26]). To test the sensitivity of the reconstruction procedure to a particular choice of the cluster algorithm, the LUCLUS and JADE algorithms were used for the PYTHIA model with the default set of parameters.

Typical uncertainties on the top-mass reconstruction are within the $\pm 180\,\mathrm{MeV}$ range, assuming a systematic off-set of $\sim 200\,\mathrm{MeV}$. The main uncertainty is due to the

[1] This has been verified by fitting the invariant mass very close to the nominal top mass

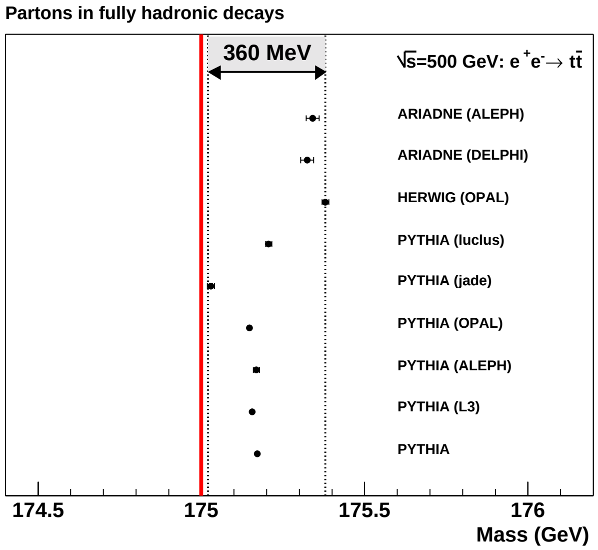

Fig. 2. The masses of top candidates in the fully hadronic $t\bar{t}$ decays. The reconstruction is performed using the parton-level MC predictions. The Durham jet algorithm was applied everywhere, except for PYTHIA (JADE) and PYTHIA (LUCLUS). For these two cases, as well as for the symbol labeled as "PYTHIA", the PYTHIA default parameters were used. The solid line indicates the nominal top mass, while the dashed lines indicate the size of uncertainties

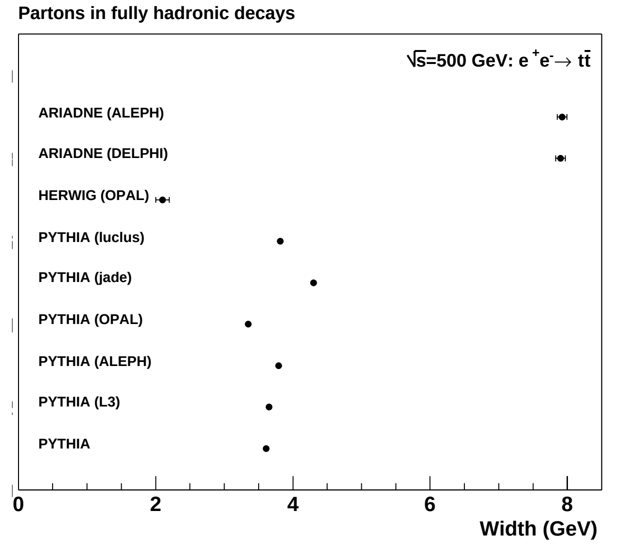

Fig. 3. The widths of the Breit–Wigner fits used to reconstruct the top candidates in the fully hadronic decays. All other details as for Fig. 2

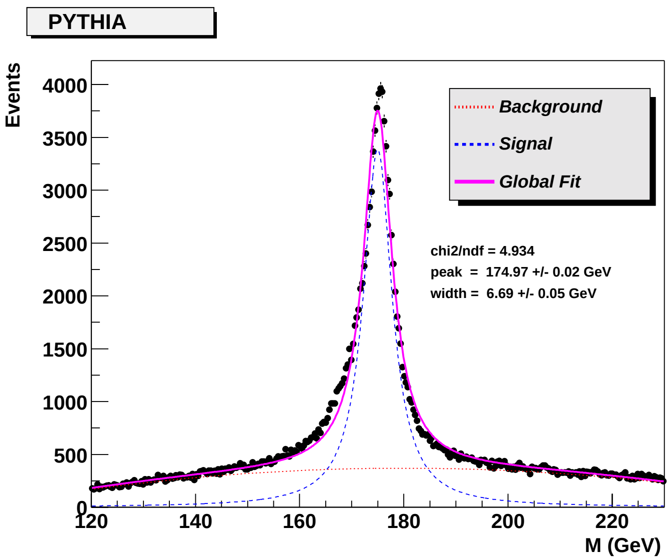

Fig. 4. The invariant-mass distribution used in the reconstruction of top candidates in the fully hadronic $t\bar{t}$ decays. The final-state particles generated with PYTHIA were used for the fits

use of the ARIADNE and HERWIG models[2], as well as due to the use of the JADE algorithm.

It is important to note that the obtained uncertainty includes not only the differences in the implementation of the high-order QCD effects by various MC models, but also uncertainties within a particular parton-shower approach. For example, the QCD cut-off, Q_0, used to terminate the partonic cascade is usually close to 1 GeV. A typical uncertainty on this value is on the level of $\pm$(15–20)%, depending on a specific tuning. Another parameter, the QCD scale in the parton-shower evolution, Λ_{LLA}, also affects the dynamic of the parton cascade and, depending on an experimental input for MC tunings, can vary by $\pm$20%.

We have not attempted to estimate the total systematical error by adding all contributions in quadrature, since the systematical uncertainties arising from Monte Carlo models with various LEP tunings are strongly correlated and cannot be combined. The best example is ARIADNE, which has a similar shift with respect to PYTHIA for all studied tunings.

[2] Note that the HERWIG top-mass distribution was treated differently than other models: since HERWIG does not contain the Breit–Wigner distribution for the generated top mass, the Breit–Wigner fit is not applicable. Therefore, the peak position and the width were determined from the mean and RMS values of the histogram defined in the mass range of 170–180 GeV

5 Reconstruction from the hadronic final state

5.1 The all-hadronic channel

The main limitations on an accurate extraction of the top mass are expected to come from the non-perturbative phase. As before, we will not freely modify tunable MC parameters, but rather will use known tunings from the LEP experiments.

The method of the top reconstruction has been discussed in Sect. 3.1. Figure 4 shows the invariant-mass distribution for the fully hadronic top decays predicted by the PYTHIA model (with the default parameters), together

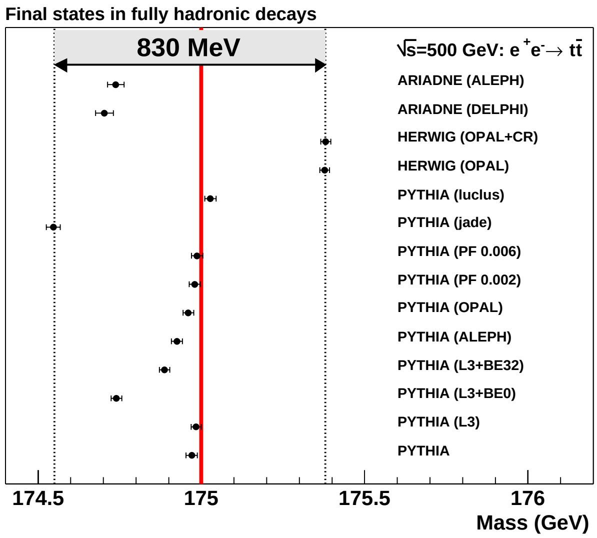

Fig. 5. The top masses in the fully $t\bar{t}$ hadronic decays reconstructed using the hadronic final state. The Durham jet algorithm is used everywhere, except for PYTHIA (jade) and PYTHIA (luclus). For these two cases, as well as for the symbol labeled as "PYTHIA", the default parameters were used. The solid line indicates the nominal mass value, while the dashed lines show the range of MC uncertainties

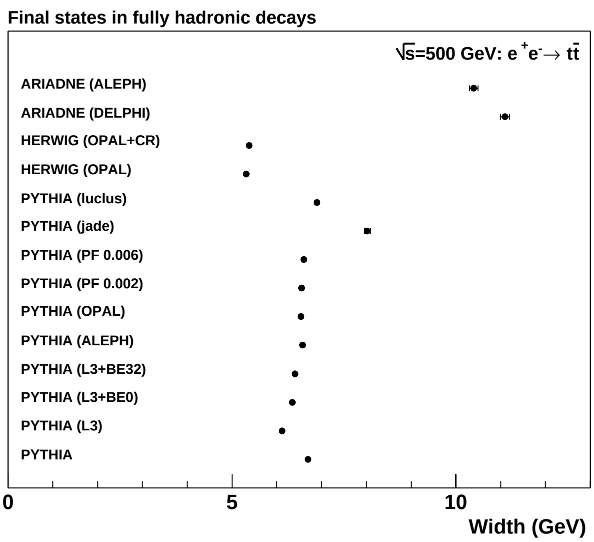

Fig. 6. The widths of the Breit–Wigner fit function obtained during the reconstruction of the top masses shown in Fig. 5. All other details as for Fig. 5

with the Breit–Wigner fit and a polynomial function for the background.

The reconstructed top masses and widths are shown in Figs. 5 and 6, respectively. As before, when algorithms were applied other than the DURHAM algorithm, the PYTHIA parameters were set to the default values. One sees an impressive stability of the results for PYTHIA with various LEP tunings. HERWIG and ARIADNE yield a comparable size of deviations from the top mass obtained from PYTHIA, but in different directions from the PYTHIA prediction. The largest systematic shifts arise from the following.

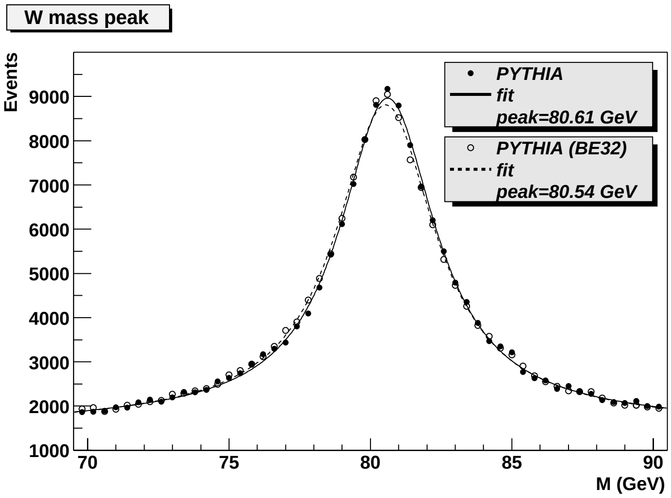

Fig. 7. The dijet invariant masses used to reconstruct the top masses in the fully hadronic $t\bar{t}$ decays. The PYTHIA model with and without the BE effect was used for the Breit–Wigner fits

(1) First, we have the choice of Monte Carlo models. The HERWIG model predicts systematically larger top masses than PYTHIA does, while ARIADNE has a shift to a smaller mass value. Since ARIADNE does not show the same feature for the parton-level studies, we conclude that the observed shift for the hadron level is due to the inclusion of the LUND string fragmentation in the color-dipole model. Note also that the ARIADNE mass spectrum is broader than the mass distributions in other MC models (Fig. 6), and this is already seen for the parton-level studies shown in Fig. 3. The top width from the HERWIG model is smaller than for PYTHIA, since HERWIG does not contain the Breit–Wigner distribution for the top decays.
(2) Next we consider the way how the BE correlations are described by MC models. The PYTHIA (L3+BE0) tuning corresponds to a model with the BE effect simulated using the global energy compensation [7]. Note that a more advanced BE modeling implemented in PYTHIA, the so-called "BE32" [7], does not show the same magnitude of the deviation. Yet, despite the fact that the BE modeling with the global energy compensation is known to be problematic, it should be noted that the PYTHIA (L3+BE0) has been tuned by the L3 Collaboration [27] to reproduce the global shape variables and single-particle densities at Z^0 peak energy. On the other hand, the model with the BE32-type of modeling was neither tuned to the global event shapes, nor to the BE correlation effect.

As was mentioned before, the BE effect can produce a systematic shift in the measurements of the W mass at LEP2. For a linear collider, it has been noted that the observation of the BE effect in $e^+e^- \to W^+W^-$ is difficult, since both W bosons are well separated kinematically for a higher center-of-mass energy than at LEP2 [28]. For the top decays, it was verified that the systematic shift after the inclusion of the BE effect comes from a smaller value of the reconstructed W mass; the shift after the inclusion

of the BE32 effect amounts to ~ 70 MeV, as illustrated in Fig. 7.
(3) Finally, a significant shift was found using the JADE algorithm.

At this moment, it is impossible to say whether the color reconnection effect can lead to an additional systematic uncertainty, since PYTHIA does not include this effect for the top production, and HERWIG does not show any sizable shift. It has to be noted that the direct reconstruction of the top mass might be uncertain by ~ 100 MeV due to the CR effect [18].

The determined systematical uncertainties for the all-hadronic channel are within ± 415 MeV range, if the JADE-type of reconstruction is included. Note again that the JADE algorithm is not as good as other algorithms for the W mass reconstruction [13]; therefore, as before, it is reasonable to quote the systematic uncertainties without use of the JADE algorithm. If the JADE is not included, the uncertainty range is reduced to ± 340 MeV.

The restriction $\mid M_W - M \mid < 5$ GeV used to select the W candidates is rather tight in practice. Moreover, such a selection is rather harmful because it affects the tails of the Breit–Wigner distribution for the reconstructed W bosons. To avoid this bias, the W candidates were selected using the following alternative method.

(1) For a given jet algorithm, covariance matrices were constructed in the three variables energy (E), polar angle (θ) and azimuthal angle (ϕ) of the initial quark. The covariance matrix elements were determined as widths of the Gaussian distributions for the $X_{\mathrm{hadrons}}/X_{\mathrm{partons}}$ variable, where $X = E, \theta, \phi$ are defined for the jets of hadrons (partons). The covariance matrix in the θ–ϕ variables was stored in a 5×5 grid, while the covariance matrix for the jet energies was calculated in 5 bins, from 10 to 170 GeV.
(2) The remaining step was to translate the covariance matrix for jets into an error on the dijet invariant mass, after a proper numerical error propagation. Then, for each dijet mass, a χ^2 value was determined from the deviations from the known nominal value of M_W. The combination which has $\chi^2 < 1$ was accepted for the top reconstruction.

Figure 8 shows the invariant-mass distribution determined from the PYTHIA model. The filled histogram shows the W candidates (having passed the $\chi^2 < 1$ restriction) used in the final reconstruction of the top quarks. Figures 9 and 10 show the values of peaks and widths for the reconstructed top quarks. In general, the obtained results are similar to those obtained using the restriction $\mid M_W - M \mid < 5$ GeV which affects the Breit–Wigner tails for the W decays. However, there exist some differences: the JADE-type reconstruction is not the dominant uncertainty anymore, and the observed uncertainty, ± 425 MeV, is due to differences between different MC models.

5.2 Semi-leptonic top decays

In the case of semi-leptonic decays, the uncertainties due to the use of different jet algorithms are expected to be small, since in this study jets are not used in the reconstruction of the W momenta.

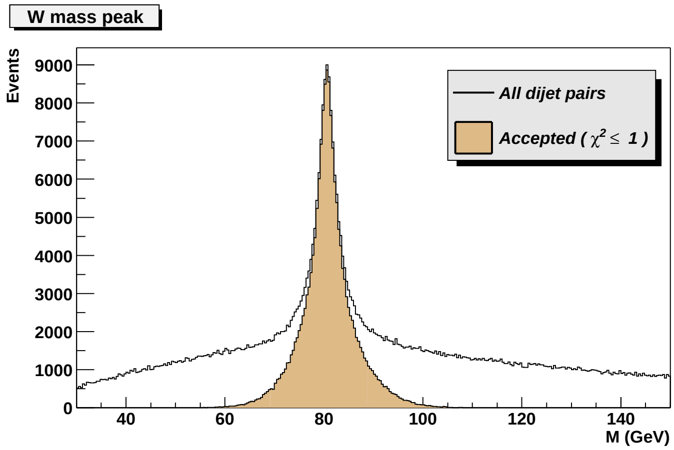

Fig. 8. The dijet invariant-mass distribution used in the reconstruction of the fully hadronic $t\bar{t}$ decays. The hatched area shows the W invariant masses used in the reconstruction of top quarks (the so-called χ^2-method)

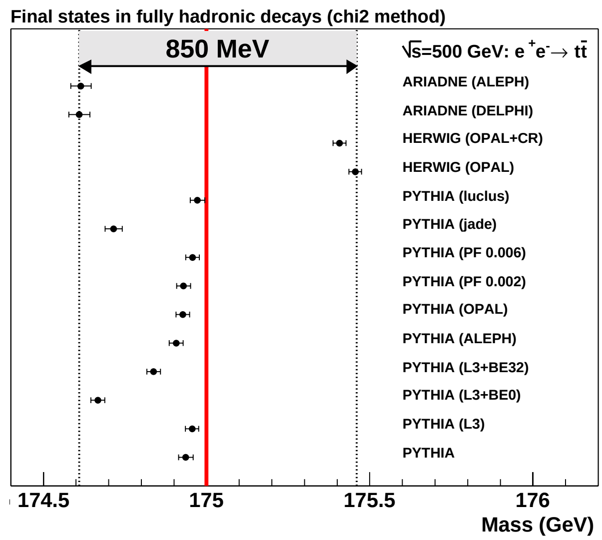

Fig. 9. The masses of top-quark candidates reconstructed in the fully hadronic $t\bar{t}$ decay. The W candidates were reconstructed from the dijet masses passed the χ^2 restriction shown in Fig. 8

Figures 11 and 12 show the reconstructed masses and widths for the same Monte Carlo models as for those used in the study of the fully hadronic $t\bar{t}$ decays. All MC uncertainties are within the ± 260 MeV range. As before, the largest uncertainty comes from the use of the JADE algorithm, applied to reconstruct the b-initialized jets, and from the use of the ARIADNE or HERWIG model. If the JADE algorithm is not used, the uncertainty range is only slightly smaller and amounts to ± 250 MeV. Note that for this decay channel the shift from the nominal mass is negligible after the inclusion of the BE32 effect. Obviously, this is because the W momenta were reconstructed intentionally without the use of hadronic jets.

For the fully hadronic $t\bar{t}$ decay, the reconstructed masses are shifted to a value smaller than the nominal top mass. These shifts are due to heavy tails of the mass

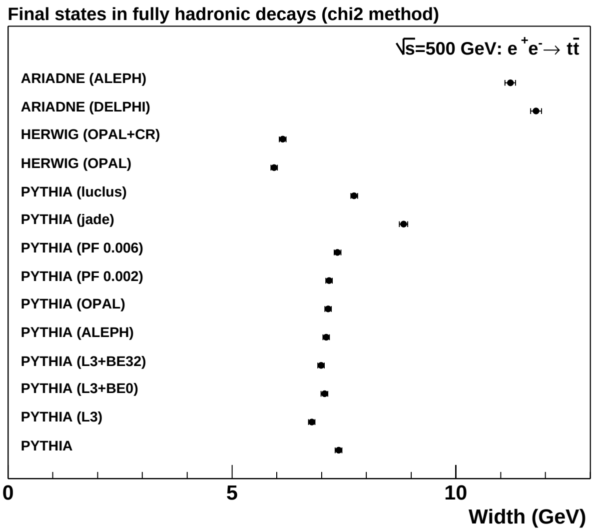

Fig. 10. The widths of the Breit–Wigner fit function used in the reconstruction of the top masses shown in Fig. 9

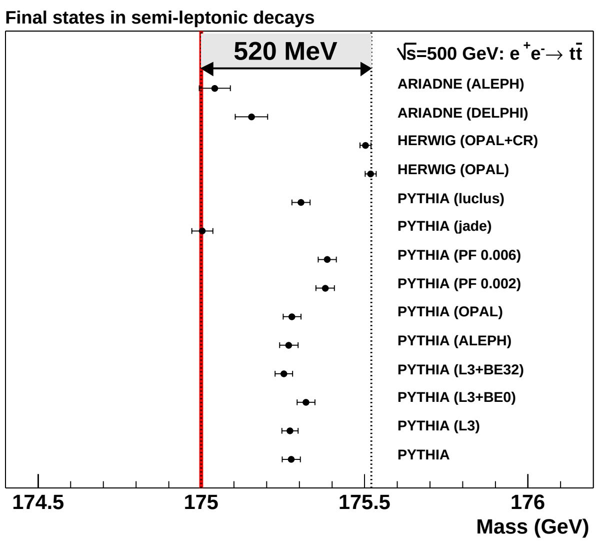

Fig. 11. The top masses reconstructed in the semi-leptonic $t\bar{t}$ decays. The W candidates were determined from a high p_{T} lepton and neutrino (calculated from the missing event momentum)

distributions from the left side of the Breit–Wigner peak, caused by contributions from unmeasured neutrinos in heavy particle decays (mainly due to charmed hadrons). In contrast, for the semi-leptonic decays, the average reconstructed mass is shifted to a value larger than the nominal mass. This is again due to the impact of neutrinos from heavy-flavored hadrons: the momenta of neutrinos from the W leptonic decays are overestimated when they are determined from the missing event momenta.

6 Summary and discussion

While the ultimate top-quark mass precision may eventually be achieved by scanning the $t\bar{t}$ production threshold, it is essential to understand the accuracy on the top-mass

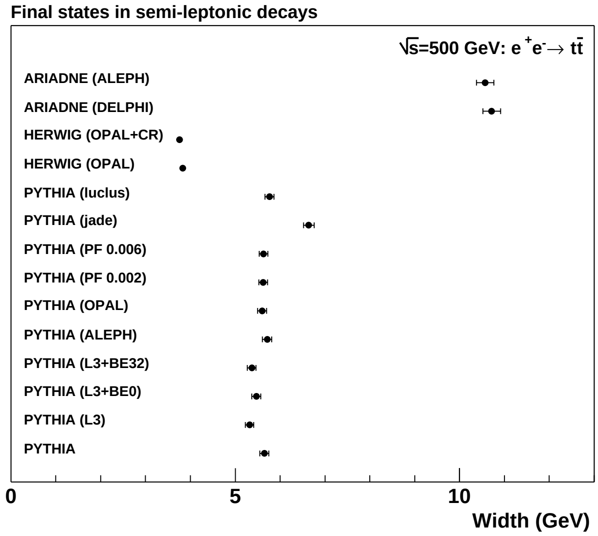

Fig. 12. The widths of the Breit–Wigner fit function used in the reconstruction of the top masses shown in Fig. 11

measurement using the direct identification of top quarks from the hadronic final state. This will require a relatively small experimental effort, will not be hampered by the lack of statistics, and will be useful for many physics topics involving the measurements of top-quark properties. As a disadvantage, the uncertainties on the reconstructed top mass can be determined for the pole mass definition, which is known with less accuracy than the top-quark $\overline{\mathrm{MS}}$ mass.

For e^+e^- colliders, the top-mass measurements will be limited by the systematical uncertainties which are tightly linked to the Monte Carlo models used to predict properties of the hadronic final state in top decays. In this paper we have estimated the uncertainties due to the current understanding of multi-hadronic final state for the top-decay channels which will be dominant at future e^+e^- colliders. Excluding the JADE-type of reconstruction, the uncertainties on the top mass in the fully hadronic decays are approximately within $\pm$(340–425) MeV range, while for the semi-leptonic decay channel this value is smaller and amounts to $\pm$250 MeV. The largest uncertainties for both decay channels are due to differences in the MC simulation of the underlying physics. For the fully hadronic top decays, the implementation of the Bose–Einstein effect between identical final-state hadrons produces an important systematic shift, ranged between 100 MeV and 250 MeV, which needs to be studied further. The results also indicate a sensitivity to the experimental methods used to extract the mass; attempts to take into account the Breit–Wigner tails of the W bosons originating from the decay $t \to Wb$ increased the systematic uncertainty for the all-hadronic top-decay channel from 340 MeV to 425 MeV.

While detailed studies remain to be carried out, it is clear that the uncertainties discussed in this paper might be reduced below the obtained values after better understanding of the multi-hadronic final state, improving the MC models, as well as after further optimization of the MC tunings by using available e^+e^- data.

We should stress again at this point that the quoted errors do not include all known sources of the uncertainties coming from the hadronic final state. First of all, some potentially important effects are missed, since they are either absent in the present versions of the MC models, or LEP tunings do not contain variations of the corresponding parameters responsible for these phenomena. For example, the color-reconnection effect was only briefly discussed in this paper due to the lack of MC modeling and tunings. Secondly, it is important to note that the quoted errors ($\pm 340/250$ MeV) do not represent *the total* theoretical uncertainties on the top-mass measurement coming from the hadronic final state, since the uncertainties from various sources studied in this paper have not been added. Therefore, the results discussed are the limits on the minimal possible uncertainties due to the hadronic final-state phenomena. The obtained numbers should be larger if there exist effects that give a larger uncertainty than any of the effects discussed in this paper. At this moment, however, it is unlikely that such effects exist. Of course, the situation is different in the case of the calculation of the total theoretical error on the top-mass measurement, for which any additional uncertainty always increases the final error. At present, to evaluate the total theoretical error on the top-mass measurement, even taking into account the effects discussed in this paper, is difficult without a proper understanding of the correlations between different contributions. Adding the uncertainties in quadrature (or linearly) usually leads to a rather pessimistic estimate; this case requires certain assumptions and a careful selection of systematic checks. Finally, uncertainties are also expected from the electroweak sector (QED initial-state photon radiations, uncertainties on the W mass determination etc.) which are usually better understood, but still need to be evaluated and properly combined with other uncertainties.

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